Consider the time evolution of the flow of a fluid (initially at rest) between two parallel plates separated by a distance $2h$. The governing Navier-Stokes equations for this flow are simplified to:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2},$$

where $dp/dx$ is a constant pressure gradient.

1. Nondimensionalize all the variables and the governing equation (including initial and boundary conditions).

2. Determine the ‘exact’ solution.

3. Find the numerical solution using (i) explicit and (ii) implicit methods.

Compare the results of your numerical solution with the exact solution. Investigate accuracy and stability by considering the effects of $\Delta t$, $\Delta y$, and $r = \Delta t/(\Delta y)^2$.

4. Find the numerical solution using a 4th-order Runge-Kutta method. Study the effect of $\Delta t$ by running simulations with various $\Delta t$.

In the presentation of results, include plots of:
- Velocity profile
- Flow rate versus time
- Wall shear versus time